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Higher Order Markov Chain Model for Synthetic Generation of Daily Streamflows

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ABSTRACT. The aim of this study is to further investigate the two-state Markov chain model for synthetic generation of daily streamflows. The model presented in [4] to determine the state of the stream and later studied in [2] and [3] is based on two Markov chains, both of order one. In some areas of Hydrology, where Markov chains of order one have been successfully used to model events such as daily rainfall, researchers are concerned about the optimal order of the Markov chain [10]. In this paper, an answer to a similar concern about the model developed in [4] is given using the Bayesian Information Criterion (BIC) to establish the order of the Markov chain which best fits the data. The methodology is applied to daily flow series from seven Brazilian sites. It is seen that the data generated using the optimal order are closer to the real data than when compared to the model proposed in [4] with the exception of two sites, which exhibit the shortest time series and are located in the driest regions.

Keywords: Bayesian Information Criterion, Hydrology, Stochastic Processes.

1 INTRODUCTION

In water resources, one of the first daily flow models was Svanidse's approach [17] in which the process was modeled based on the definition of a fragment. The fragment is a set of annual daily flows sequences obtained by dividing the daily flows of the considered year by the mean annual flow of the same year. The procedure can be described by (a) the generation of the mean annual flow value; (b) random drawing with replacement of one element of the fragment set; and (c) multiplication of every member of the fragment by the mean annual flow value. Besides the over-reliance on the mean flow, Svanidse's model has the additional shortcoming of assuming

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independence between the mean annual flow and properties of daily flows within the annual period.

Soon after the proposal of Svanidse's model, autoregressive models were used for simulation of daily flows [6, 13]. In [6], the mean monthly flows are generated first and, in a next step, they are dismembered into daily values using a second-order autoregressive model, AR(2). In [13], the AR(2) model is directly used to simulate daily flow. In both cases, statistical characteristics of the AR(2) process are estimated over time. These AR(2) models cannot reproduce recessions because of the underlying white noise processes.

Later, a model to generate daily streamflow based on linear interpolation of five-day average flows using statistical modeling for the non-deterministic component of the daily time series was proposed by [9]. However, the model masks short-term fluctuations, an important feature in daily streamflow. Non-parametric techniques were used in [12]. The advantage is that non-parametric methods do not require distributional specifications needed by parametric methods. Generated streamflow series using this technique may retain the marginal and joint density structure of the observed hydrologic series including nonlinearity and state dependence. Seasonality in the daily flow process was modeled by [18] by assuming a periodic structure, within an annual cycle, of the daily mean and variance. At the same time, he assumed that the system's response function was invariant accross the annual cycle, stating that this assumption was often made but only quoting a past writing [20] in which there are no grounds for such assumption.

One of the objectives in stochastic hydrology is to generate synthetic streamflow sequences that are statistically similar to observed data. Statistical similarity implies that the generated sequences have statistical and dependence properties similar to those of the historical record. In fact, autoregressive moving average models, ARMA(p,q), together with the Fractional Gaussian Noise (FGN) model and variations dominate streamflow generation [19]. Several of these models have been applied to daily streamflow generation. However, ARMA(p,q) recessions, which are sums of negative exponential functions, could not account for the prominent features of daily streamflow. Even when seasonality is considered (SARIMA), such models cannot capture the peculiarities of daily flow data.

Markov chains have also been used as an important tool in the studies of hydrometeorological variables at a daily time interval [14]. Some seminal works include [15] and extensions [4, 5]. These extended models consist of four steps: (i) determination of the days in which flow occurs, (ii) determination of the days in which a flow increment occurs, (iii) determination of the flow increment, and (iv) calculation of the flow decrement on days when the flow is reduced. In [5], the first two steps are modeled by a three-state Markov chain and in [4] these steps are modeled by two two-state Markov chains. The applicability of both techniques is showed in [2] and their performances can be seen in [3] where the conclusion is that both alternatives are capable of simulating the state of the stream.

In accordance with [10], although Markov chains of order one have been successfully employed to describe the occurrence of daily rainfall, there remains uncertainty concerning the optimal order for stremflows. Our paper is concerned with the estimation of the optimal order of Markov chains used in [4] for generation of daily streamflows. In particular, the Bayesian Information Criteria (BIC) is used to determine the order that better fits a given set of data. We employ this technique to data from seven Brazilian sites. After estimating the optimal order, we fit the corresponding Markov chain and use it for synthetic generation of daily streamflow, thereby providing basic hydrologic data for integrated water resources management in the sites considered.

2 METHODOLOGY

The basic model employed here is based on the following steps described in [4] and [2]: for each month we determine (i) days in which flow occurs, (ii) days in which a flow increment occurs, (iii) the flow increment/decrement. The two first steps are modeled by two Markov chains. At step (i), a 1 - 0 Markov chain of order one is used to assign 1 for the occurrence of flow and 0 for the non-occurrence of flow:

$$\begin{bmatrix} P_{11} & P_{10} \\ P_{01} & P_{00} \end{bmatrix}, \tag{2.1}$$

where P_{ij} denotes the probability that the system makes a (one-step) transition to state j given that it is at state i and the resulting 2×2 matrix represents the transition matrix of the two-state Markov chain modeling flow occurrence. Once a day with flow has been determined, step (ii) uses another two-state Markov chain of order one to choose between an increment (R) or a decrement (F) of flow for that day:

$$\begin{bmatrix} P_{RR} & P_{RF} \\ P_{FR} & P_{FF} \end{bmatrix}, \tag{2.2}$$

where R corresponds to a day with a flow increment (rise) and F corresponds to a day with a flow decrement (fall).

In this paper, we modify the basic model described above by letting the order of the Markov chains used in steps (i) and (ii) be determined by the value of the BIC, estimated from the data.

2.1 Determination of the order of the Markov chains based on information criteria

Akaike's Information Criterion (AIC) is the standard metric for model comparison in several areas of data analysis, notably in time series and stochastic processes [7]. However, it is not consistent to estimate the order of a Markov chain based on the asymptotic distribution of the resulting estimator [11]. Looking for estimators that have better properties, [11] has suggested the Bayesian Information Criterion (BIC) [16] as an alternative to the AIC. The order β is estimated by $\hat{\beta}$ which is the value that minimizes the BIC across the models being entertained:

$$\mathrm{BIC}(\hat{\beta}) = \min\{\mathrm{BIC}(\beta), \beta = 0, 1, 2, 3, \ldots\}. \tag{2.3}$$

Consider a Markov chain of order β with N states such that $n_{i_1,i_2,\ldots,i_\beta}$ is the number of times that $(i_1,i_2,\ldots,i_\beta)$ appears in a sample of size n. Then

$$\mathrm{BIC}(\beta) = -2 \sum_{i_1,i_2,\ldots,i_{\beta+1}=1}^{N} n_{i_1,i_2,\ldots,i_{\beta+1}} \log\left(\frac{n_{i_1,i_2,\ldots,i_{\beta+1}}}{n_{i_1,i_2,\ldots,i_\beta}}\right) + \gamma(\beta)\log(n), \tag{2.4}$$

where $\gamma(\beta) = N^{\beta}(N-1)$ is the number of free parameters under the hypothesis that the order is β. Strong consistency of the BIC-derived estimator has been proved and extended for cases where finiteness of the order is not assumed [8].

Here, for each month, the BIC is used to estimate the order of the chain that best models flow/no flow and of the chain for increment/decrement. Thus, if the BIC of the raise/fall chain of a given month indicates that the order is zero, the values of the increment or decrement of the samples are obtained independently using the rate of climbs as success probability. If the order is one, the transition matrices flow/no flow and increment/decrement reduce to (2.1) and (2.2) respectively.

When the order of the Markov chain is two, the current state receives two pieces of information, the first information at time t and the second information at time $t+1$ (for rises and falls, for example, the possible states are: RR, RF, FR, and FF). Thus, the first pair of information refers to times t and $t+1$ and the second pair to times $t+1$, $t+2$. Generally speaking, in a Markov chain $\{x_t\}_{t\in N}$ of order two, the transition matrix is given by

$$P(AB,CD) = P(x_{t+2} = D, x_{t+1} = C | x_{t+1} = B, x_t = A), \tag{2.5}$$

where A, B, C, D are states of the chain and its transition matrix is estimated by

$$\hat{P}(AB,CD) = \begin{cases} \frac{n_{ABD}}{n_{AB\bullet}}, & \text{if } n_{AB\bullet} \neq 0, \quad B = C, \\ 0, & \text{if } n_{AB\bullet} \neq 0, \quad B \neq C, \\ \delta_{AB}(CD), & \text{if } n_{AB\bullet} = 0, \end{cases}$$

where n_{ABD} is the number of ABD transitions that appear in the sample, $n_{AB\bullet}$ is the number of ABT triples where T is any of the possible states of the chain, and $\delta_{AB}(CD)$ is the Kronecker delta, which is equal to 1 when $AB = CD$ ($A = C$ and $B = D$) and zero otherwise.

The principle of model parsimony [7] dictates that simpler (having fewer parameters) models should be selected whenever more than one model fits the data well. In practice, the trade-off between goodness of fit and parsimony must be measured by some metric such as the BIC for model selection. In the BIC formulation, its value is directly proportional to k (the number of parameters) and higher orders will be chosen over more parsimonious, lower order, models only if the goodness of fit is improved.

2.2 Flow increment and flow decrement

The flow increment is the difference between successive daily flows when the flow of the current day is greater than the flow of the previous day. Based on data, and following [4, 2, 1], the flow increment is modeled with a two-parameter gamma distribution with density function

$$f(x) = \frac{\lambda^{\alpha} x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}, \quad x > 0, \tag{2.6}$$

where α and λ are the shape and scale parameters, respectively, and Γ denotes the gamma function. The expected value $\mathrm{E}[X]$ and variance $\mathrm{Var}[X]$ of the two-parameter gamma distribution are

$$\mathrm{E}(X) = \frac{\alpha}{\lambda} \quad \text{and} \quad \mathrm{Var}(X) = \frac{\alpha}{\lambda^2}. \tag{2.7}$$

Given daily observed historical series, the expectation and the variance above are replaced by the corresponding monthly sample statistics, so the shape (α) and scale (λ) parameters of the distribution are estimated for each month,

$$\hat{\alpha} = \left(\frac{\bar{x}}{S}\right)^2 \quad \text{and} \quad \hat{\lambda} = \frac{\bar{x}}{S^2},$$

where $\bar{x}$ and S are the sample mean and standard deviation, respectively, for a given month.

The total number of parameters required for this step of the model is, thus, 24 for each of the seven sites analyzed in Section 3 below.

In order to determine the flow decrement, which occurs when the flow of the current day is less than the flow of the previous day, the data are divided in two sets:

- days in which the decrement happened when the flow of the previous day is greater than the monthly mean flow.
- days in which the decrement happened when the flow of the previous day is less than the monthly mean flow.

The values of the first set are used to estimate the parameter b_1, the flow decrement rate in one day. Once again, following [4, 2], the parameter b_1 models the flow decrement when the flow of the previous day (Q_{t-1}) is greater than the monthly mean flow by means of

$$Q_t = Q_{t-1} e^{-b_1}. \tag{2.8}$$

In a similar way, the values of the second set are used to estimate the parameter b_2, that models the flow decrement when the flow of the previous day (Q_{t-1}) is less than the monthly mean flow by the equation:

$$Q_t = Q_{t-1} e^{-b_2}. \tag{2.9}$$

For each month of the year, the parameters b_1 and b_2 and the monthly mean flow value are determined from the observed data. The parameters are estimated by least squares after linearization of (2.8) and (2.9): $\log Q_t = -b_i + \log Q_{t-1}$, $i = 1,2$. Thus, 24 parameters are estimated in this step.

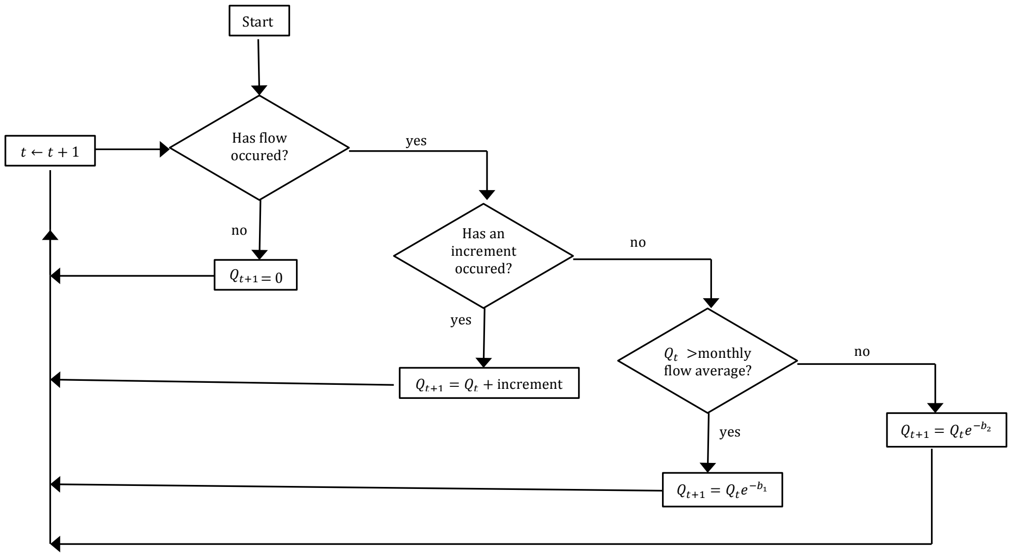


Figure 1: Flow chart.

2.3 The flow chart

Using the values of all parameters previously calculated and beginning with a flow value that is randomly generated from the set of all values observed in the first day of the historical data (usually Jan 1^{st}) the generated streamflow is obtained as given in the flow chart in Figure 1.

When a new month is reached, parameters are automatically updated to the corresponding month, and the process continues. As used in Aksoy's model, this process is repeated 10 times. Each period should have a length of 10 times the number of years of data used, that is, if the size of the data sample is 10 years, each realization should be run for 100 years.

3 HISTORICAL DATA USED

The developed model was tested using daily data of seven sites across the Brazilian territory in different periods and climates. The sites are: Carolina, Campo Largo, Porto do Lopes, Fazenda Ajudas, Ribeiro Gonçalves, Fazenda da Barra and Alberto Flores. The stream gauges are between 122 and 725 m above the mean sea level. The drainage areas of those stream gauges varied from 244 to 275,000 km^2. The average long term flow varied from 5.2 to 3790.9 m^3/s and the sample sizes varied from 35 to 60 years. The data were obtained from the Brazilian National Water Agency (ANA) (`www.ana.gov.br`). Table 1 summarizes this information.

Table 1: Basic information on gauging stations used in the analysis. † NA/NE refers to North Atlantic/Northeast basin.

Stream gauge	State	Latitude	Longitude	Elev. (m)	River basin	Drainage area (km^2)	Lenght (years)
Carolina	MA	7º20′15″S	47º28′23″W	122	Tocantins	275,000	46
Campo Largo	MA	6º04′01″ S	44º42′30″W	204	NA/NE†	5,750	35
Porto do Lopes	MA	6º00′26″S	44º20′24″W	150	NA/NE	6,890	36
Ribeiro Gonçalves	PI	7º34′00″S	45º15′16″W	150	NA/NE	31,300	40
Fazenda da Barra	MG	20º12′56″S	46º13′56″W	668	São Francisco	757	42
Fazenda Ajudas	MG	20º5′45″S	46º03′51″W	681	São Francisco	244	60
Alberto Flores	MG	20º09′25″S	44º10′00″W	725	São Francisco	4,120	43

4 RESULTS

4.1 Illustration with the Carolina river data

The Carolina River is used next to illustrate the procedure proposed here to estimate the order of the Markov chain that modeled the increment/decrement flow. In the sequel, the estimated order given by the BIC analysis for each river is presented as well as the parameters of the functions that modeled the flow increment and the flow decrement.

Using eq. (2.4) with data from the Carolina River, the values of the BIC for each month are shown in Table 2.

Table 2: BIC values for rise/fall - Carolina River Data; minimum BIC in bold.

Month	β					
	0	1	2	3	4	5
January	1919	1353	1317	1299	**1294**	1352
February	1677	1169	1144	**1103**	1114	1168
March	1857	1289	1267	**1242**	1253	1308
April	1711	1257	1211	1184	**1181**	1236
May	1163	872	847	**812**	823	878
June	909	679	658	**648**	671	729
July	945	658	**640**	645	646	697
August	1239	842	821	**799**	821	889
September	1723	1186	1151	1130	**1127**	1191
October	1876	1462	1429	**1407**	1422	1467
November	1789	1392	1354	1323	**1316**	1372
December	1912	1387	1345	**1308**	1312	1361

The chosen order is the one yielding the smallest BIC. From Table 2, the optimal order β for the rise/fall matrices for the Carolina River are given in Table 3.

Table 3: Optimal order of rise/fall chains for each month - Carolina River.

	Jan	Fev	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
β	4	3	3	4	3	3	2	3	4	3	4	3

4.2 Summary of results from all rivers

Implementing the same scheme illustrated above for the Carolina River to the other six sites yields the results in Table 4. Note that the chosen order was always greater than one.

Table 4: Optimal order of rise/fall chains, for each month, obtained from analyzing the BIC values for the seven sites used in the analysis.

Site	Optimal β											
	Jan	Fev	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Alberto Flores	3	3	4	3	3	3	3	3	4	4	3	4
Campo Largo	3	3	4	4	4	2	2	2	3	3	3	3
Carolina	4	3	3	4	3	3	2	3	4	3	4	3
Fazenda Ajudas	4	4	4	3	3	3	2	2	4	3	3	4
Fazenda da Barra	4	4	4	3	4	3	3	2	3	3	4	4
Porto do Lopes	3	3	3	3	3	3	2	2	3	3	3	3
Ribeiro Gonçalves	4	4	3	3	3	2	2	2	3	3	4	4

The flow generation scheme proposed in [4], using Markov chains of order one, reproduced the main features of daily flows quite successfully.

Next, we compare the results from the scheme proposed here (MM), using Markov chains with estimated order (via BIC), with the results obtained from the model presented in [4] (MA) for each of the seven stream gauges. Tables 5–11 show streamflow values generated by MM and those generated by MA. In the last line of each table, the distance between the observed data and each generated data are displayed. The metric used to calculate such distance is

$$\|OS - GS\| = \sqrt{\sum_{i=1}^{12} [OS(i) - GS(i)]^2},$$

where i indexes month, OS represents the observed streamflow and GS represents the generated streamflows. The metric $\|OS - GS\|$ favors MM over MA for five of the sites. The exceptions are Campo Largo and Porto Lopes for which $GS_{\mathrm{MA}} < GS_{\mathrm{MM}}$.

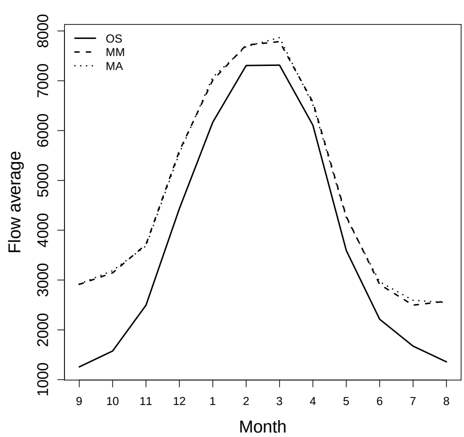
Figures 2–8 summarize the information in Tables 5–11. For each river, an average flow plot as well as estimated regression lines of the simulated versus observed data are represented for both modeling strategies. In Figures 2–8 we verify that the generated sequence from MM is closer to the observed monthly data in five sites, the exceptions being Campo Largo and Porto Lopes. The values of GS for both schemes, MM and MA, are close to each other, especially around the peak month. In most cases, both schemes tend to overestimate the monthly flow with *GS* curves almost entirely above the *OS* curve. This cannot be due to faulty choice of the chain order since both models differ in that respect (MA fixed at order one and MM based on the BIC). We believe that the flow increment/decrement scheme described in Section 2.2, which is used in both models, is the source of the problem.

Table 5: Observed (*OS*) and generated (*GS*) streamflows for the two models considered – Carolina site.

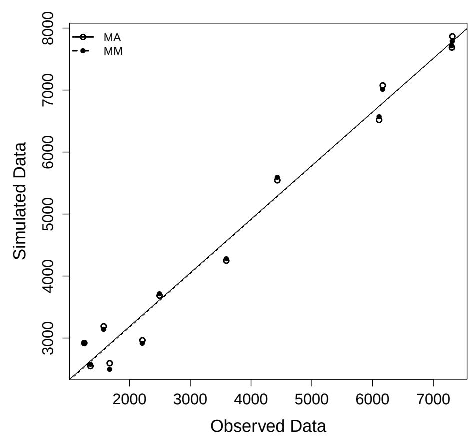
	OS	*GS*	
		MM	MA
January	6168	7012	7075
February	7306	7716	7686
March	7313	7786	7866
April	6108	6571	6518
May	3593	4275	4248
June	2213	2913	2964
July	1674	2496	2594
August	1357	2575	2548
September	1256	2912	2920
October	1575	3144	3189
November	2495	3713	3684
December	4433	5592	5545
$\|\|OS-GS\|\|$		3530	3567

Table 6: Observed (*OS*) and generated (*GS*) streamflows for the two models considered – Campo Largo site.

	OS	*GS*	
		MM	MA
January	34.33	37.19	36.61
February	37.03	39.14	38.31
March	39.27	41.73	41.28
April	40.00	42.28	41.82
May	36.63	36.50	37.12
June	32.89	30.29	31.25
July	30.73	26.66	27.10
August	29.29	25.25	24.75
September	28.73	25.92	25.21
October	29.21	27.95	27.54
November	30.18	31.04	31.38
December	31.75	34.48	34.68
$\|\|OS-GS\|\|$		9.01	8.72

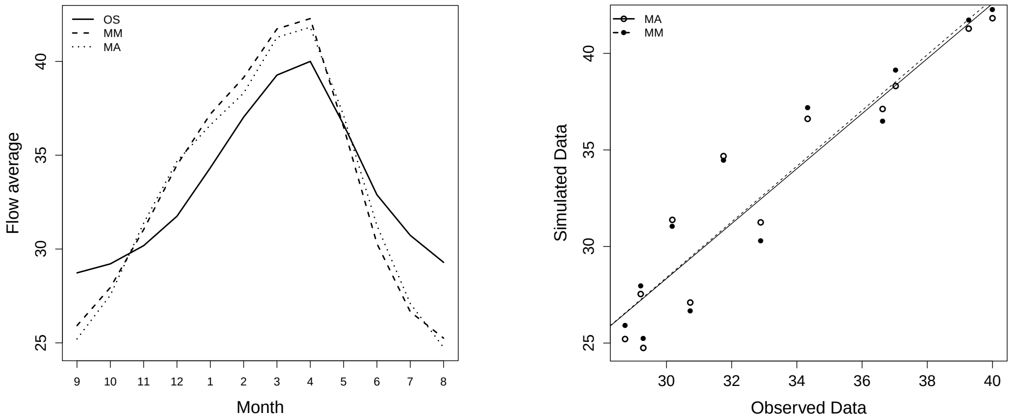


(a) Observed and generated monthly flow.



(b) Linear Regression
MA: $GS = 1457 + 0.865OS$
MM: $GS = 1439 + 0.867OS$

Figure 2: Comparison of monthly observed and generated flows – Carolina River.



(a) Observed and generated monthly flow.

(b) Linear Regression
MA: $GS = -14.4 + 1.42OS$
MM: $GS = -14.9 + 1.44OS$

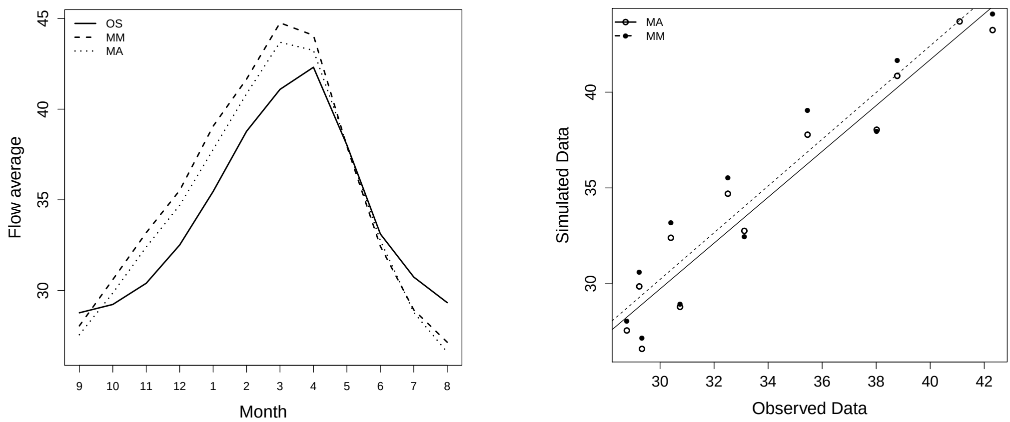
Figure 3: Comparison of monthly observed and generated flows – Campo Largo.

Table 7: Observed (OS) and generated (GS) streamflows for the two models considered – Porto do Lopes site.

	OS	GS	
		MM	MA
January	35.46	39.06	37.78
February	38.78	41.66	40.85
March	41.09	44.76	43.69
April	42.31	44.07	43.24
May	38.02	37.95	38.04
June	33.12	32.44	32.76
July	30.74	28.93	28.79
August	29.33	27.15	26.60
September	28.77	28.05	27.56
October	29.23	30.60	29.86
November	30.40	33.19	32.40
December	32.51	35.52	34.70
$\|OS - GS\|$		8.08	6.26

Table 8: Observed (OS) and generated (GS) streamflows for the two models considered – Ribeiro Gonçalves site.

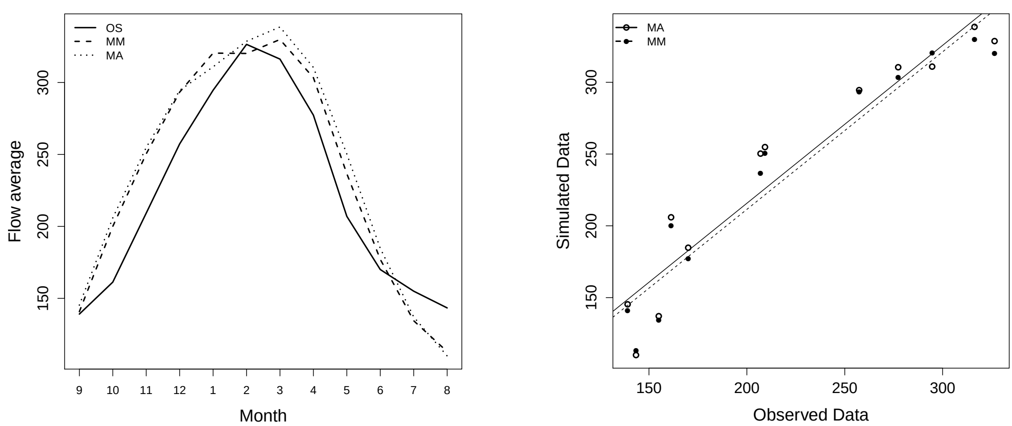
	OS	GS	
		MM	MA
January	294.7	320.4	310.9
February	326.5	320.2	328.7
March	316.3	329.9	338.6
April	277.3	303.4	310.5
May	207.0	236.6	250.3
June	170.0	177.1	184.8
July	155.0	134.5	137.1
August	143.4	113.0	110.0
September	139.1	140.8	145.3
October	161.3	200.0	205.9
November	209.3	250.5	254.9
December	257.4	293.4	294.5
$\|OS - GS\|$		91.29	104.37



(a) Observed and generated monthly flow.

(b) Linear Regression
MA: $GS = -6.15 + 1.20OS$
MM: $GS = -6.34 + 1.22OS$

Figure 4: Comparison of monthly observed and generated flows – Porto Lopes.



(a) Observed and generated monthly flow.

(b) Linear Regression
MA: $GS = -4.32 + 1.10OS$
MM: $GS = -8.05 + 1.01OS$

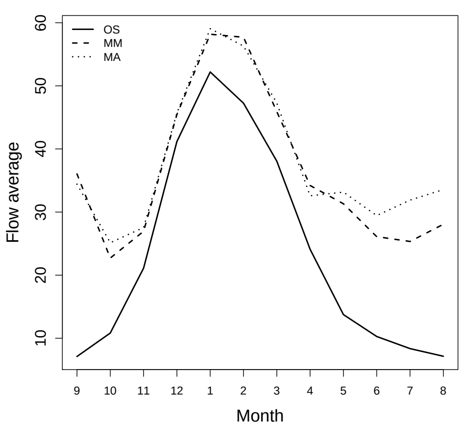
Figure 5: Comparison of monthly observed and generated flows – Ribeiro Gonçalves.

Table 9: Observed (OS) and generated (GS) streamflows for the two models considered – Fazenda da Barra site.

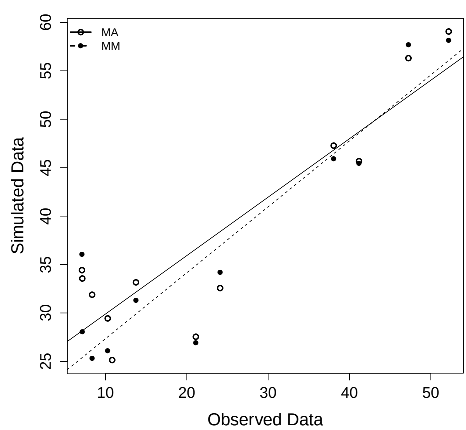
	OS	GS	
		MM	MA
January	52.20	58.18	59.06
February	47.25	57.70	56.31
March	38.06	45.91	47.28
April	24.10	34.21	32.57
May	13.75	31.31	33.16
June	10.28	26.10	29.44
July	8.36	25.34	31.89
August	7.15	28.06	33.56
September	7.11	36.04	34.42
October	10.83	22.71	25.14
November	21.11	26.95	27.55
December	41.17	45.45	45.67
$\|OS-GS\|$		51.21	57.38

Table 10: Observed (OS) and generated (GS) streamflows for the two models considered – Fazenda Ajudas site.

	OS	GS	
		MM	MA
January	11.47	12.96	13.17
February	9.90	10.71	10.58
March	8.81	10.02	10.11
April	6.24	7.40	7.85
May	3.92	6.80	7.13
June	2.95	5.64	6.09
July	2.38	4.78	4.66
August	1.89	4.66	4.83
September	1.82	5.22	5.34
October	2.13	4.82	5.07
November	3.61	6.00	5.8
December	7.27	9.68	9.78
$\|OS-GS\|$		8.06	8.60

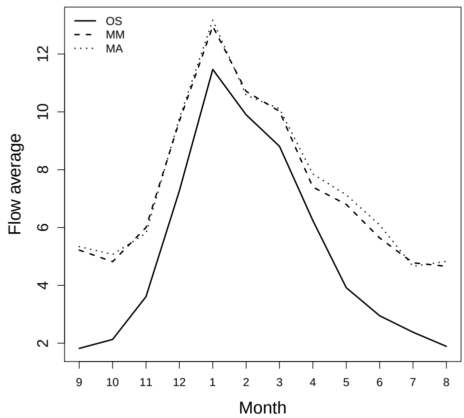


(a) Observed and generated monthly flow.

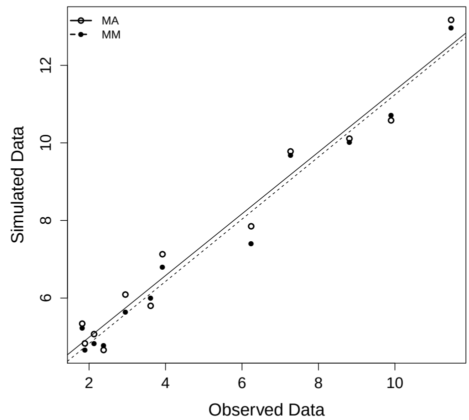


(b) Linear Regression
MA: $GS = 23.9 + 0.60OS$
MM: $GS = 20.5 + 0.68OS$

Figure 6: Comparison of monthly observed and generated flows – Fazenda da Barra.



(a) Observed and generated monthly flow.

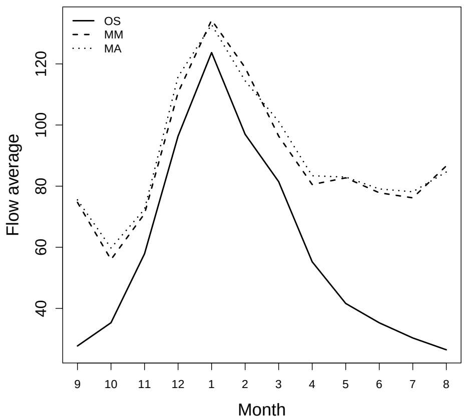


(b) Linear Regression
MA: $GS = 3.40 + 0.79OS$
MM: $GS = 3.22 + 0.80OS$

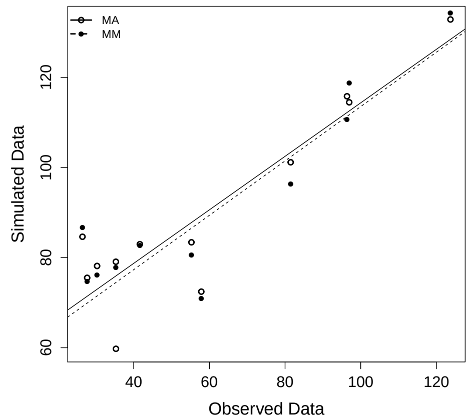
Figure 7: Comparison of monthly observed and generated flows – Fazenda Ajuda.

Table 11: Observed (OS) and generated (GS) streamflows for the two models considered – Alberto Flores site.

	OS	GS	
		MM	MA
January	123.71	134.33	132.85
February	96.96	118.78	114.46
March	81.48	96.38	101.16
April	55.26	80.54	83.40
May	41.62	82.72	82.98
June	35.29	77.84	79.09
July	30.33	76.17	78.15
August	26.46	86.63	84.63
September	27.71	74.69	75.52
October	35.31	55.98	59.77
November	57.87	70.961	72.45
December	96.37	110.61	115.8
$\|OS - GS\|$		116.95	119.77



(a) Observed and generated monthly flow.



(b) Linear Regression
MA: $GS = 55.0 + 0.59OS$
MM: $GS = 53.2 + 0.60OS$

Figure 8: Comparison of monthly observed and generated flows – Alberto Flores.

5 CONCLUSION

The use of a variable order for the Markov chains modeling streamflow data is a viable and simple feature, which can be incorporated into any modeling scheme in stochastic hydrology based on Markov chains. Here, we have done so by incorporating higher order chains into a scheme previously used to model streamflow data [4, 2, 3], which is based on Markov chains of order one. Such scheme was used to describe the occurrence of daily streamflow followed by stochastic modeling (via Gamma distribution) of flow increment and a simple exponential fit to model flow decrement. Based on the distance between the observed and generated (monthly) average flows, the results using higher order chains were better than using order one, with the exception of two studied cases, which have the shortest time series and are located in the driest region of Maranhão state. The proposed criterion for choosing the order has been the BIC.

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RESUMO. O objetivo deste estudo é melhorar o modelo de cadeias de Markov de dois estados usado em Hidrologia para geração sintética de fluxos diários. O modelo apresentado em [4] e estudado em [2] e [3] baseia-se em duas cadeias de Markov, ambas de ordem um, para a determinação do estado do fluxo. Em algumas áreas da Hidrologia, onde cadeias de Markov de ordem um são usadas com sucesso para modelar eventos como precipitação diária, pesquisadores têm se mostrado preocupados com a ordem ótima de tais cadeias [10]. Neste artigo, uma resposta a uma preocupação similar sobre o modelo desenvolvido em [4] é dada, usando o critério de informação de Bayes para estabelecer a ordem de cadeia de Markov que melhor se encaixa nos dados. A metodologia é aplicada a uma série de fluxos diários de sete rios brasileiros. Observa-se que os dados gerados usando a ordem estimada de cada cadeia são mais próximos dos dados reais do que o modelo proposto em [4], com exceção de dois locais que têm as menores séries temporais e estão localizados nas regiões mais secas.

Palavras-chave: Hidrologia, processos estocásticos, critério de informação bayesiano.

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